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HEAT TRANSFER AHEAD OF MOVING CONDENSATION FRONTS IN THERMAL OIL RECOVERY PROCESSES

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Abstract—This is a study of heat transfer in the liquid zone preceding an advancing condensation front in a 1-dim. oil reservoir undergoing a thermal recovery process such as hot waterflood, steam injection or *in situ* combustion. A model is developed that allows for heat transfer by horizontal conduction and convection in the reservoir and by vertical (conjugate) conduction in the surrounding the reservoir formations to account for the lateral heat losses. The model is formulated in terms of an integro-differential equation involving an integral representation of the lateral heat losses for a region with a moving boundary. According to the magnitude of the Peclet number, and the velocity of the advancing front, various analytical expressions that describe the temperature distribution in the hot liquid zone are derived. The discussion emphasizes the cases $Pe \gg 1$ (high injection rates) and $Pe = 1$ (low injection rates). The case of arbitrary Pe is treated by a quasi-steady state approximation

NOMENCLATURE

- a, parameter of front velocity [dimensionless];
- c, velocity of moving front [dimensionless];
- C, volumetric heat capacity [kg m^{-1} s^{-1} °C^{-1}];
- c_p, heat capacity under constant pressure [m^2 s^{-2} °C^{-1}];
- h, reservoir thickness [m];
- k, thermal conductivity [kg m s^{-3} °C^{-1}];
- Pe, Peclet number [dimensionless];
- r, coordinate along the radial direction [m];
- t, time [s];
- T, temperature;
- U, volumetric flow rate [kg s^{-3} °C^{-1}];
- u, flow velocity [m s^{-1}];
- v, front velocity [m s^{-1}]/[m^2 s^{-1}];
- x, Cartesian coordinate [m];
- z, Cartesian coordinate [m];
- z_i, root of equation (34), dimensionless;

Greek symbols

- α_i, thermal diffusivity of species i [m^2 s^{-1}];
- θ, dimensionless time;
- Θ, dimensionless temperature;
- λ, parameter defined by equation (31) [dimensionless];
- μ, parameter defined by equation (31) [dimensionless];
- ξ, dimensionless space coordinate;
- ρ, density [kg m^{-3}];
- ϕ, porosity [dimensionless];
- χ, dimensionless space coordinate;
- ω, dimensionless velocity.

Subscripts

- i, initial;
- l, refers to surrounding formations;
- o, oil;
- r, reservoir;
- s, steam;
- w, water;
- x, refers to the Cartesian coordinate x.

Superscripts

- indicates dimensional quantities.

INTRODUCTION

AN IMPORTANT class of petroleum recovery processes, e.g. steam injection and *in situ* combustion, involve the propagation of condensation fronts in the porous reservoir formation. In such processes the flow field consists of two regions separated by the condensation front—a region occupied by steam (steam zone), and a region occupied by the displaced liquids, petroleum and water (hot liquid zone). Both regions are bounded by rock formations of infinite extent which conduct heat but are impermeable to fluid flow.

The determination of the velocity of the condensation front, which is of primary importance to the economics of the process, largely depends upon the thermal losses to the surroundings and the heat distributions in the steam and the hot liquid zones. Heat transport in the hot liquid region is of particular interest since it influences the overall heat distribution in a two-fold manner: directly through the amount of heat transferred across the front and subsequently stored in the reservoir or lost to the surroundings; and indirectly, through the amount of lateral heat losses from the steam zone, the magnitude of which depends upon the preheating of the rock by the liquid zone.

Solutions to the heat transfer problem in the hot liquid zone have been obtained by means of detailed numerical schemes, [1]. While such numerical sol-

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utions are of wide scope, in many applications a less detailed but simpler analytical approach would be desirable. For example, engineering type calculations and parametric studies do not warrant lengthy and expensive computer calculations especially when reservoir properties such as geometry and permeability are not known in detail. However, in most of the existing analytical studies the heat transfer in the hot liquid zone has been inadequately treated [2], or completely ignored [3].

The present study attempts a more complete analytical treatment of heat transfer in the hot liquid zone preceding the condensation front. To this end, the heat transfer problem in the liquid zone is uncoupled from that in the steam zone by assuming constant temperature at the front and by treating the velocity of the front and the fluid flux through the front as known quantities. The first assumption is a common occurrence in most applications, while the second is justifiable for processes at constant injection rates at the early and late stages of the process (see [4] for a detailed discussion). With the aid of these simplifications the problem is formulated in terms of two coupled partial differential equations, one for the conductive and convective transport in the liquid region and one for the pure conduction in the surrounding formations. Analytical treatments of such coupled equations have been previously provided only for the case of fixed boundaries [5, 6]. The main contribution of the present paper is to include the moving boundary represented by the condensation front. In approaching this more difficult problem, the two partial differential equations are combined into a single integro-differential equation still involving a moving boundary. This equation is then solved in several special cases, some of which are of direct practical interest. The obtained solutions coupled to integral balances across the condensation zone can be further utilized in order to determine the velocity of the front in several cases (see [4]).

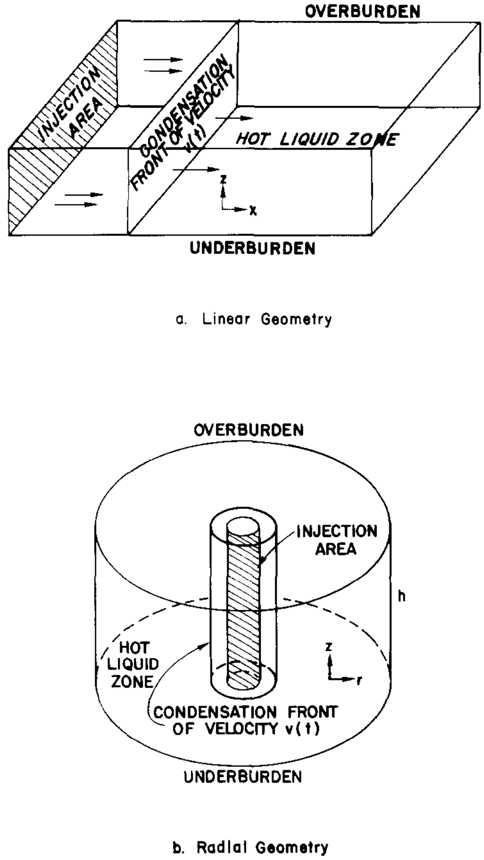

FIG. 1. One-dimensional geometries examined in section 1.

1. MATHEMATICAL FORMULATION

Consider a 1-dim. reservoir of thickness h bounded from above and below by impermeable rock strata (Fig. 1). The reservoir is initially saturated by oil and water at the initial formation temperature, T_i. At time $t = 0$, by virtue of steam injection, combustion or some other thermal process, a condensation front of constant temperature T_s develops at the origin and starts propagating inside the reservoir, which is thereby divided into a region of constant temperature and a zone of varying temperature (the hot liquid zone) (Fig. 1). At any stage during the process oil and water flow continuously through the moving front and inside the hot liquid zone, while heat flows by horizontal convection and conduction in the reservoir and by vertical conduction to the surroundings (lateral heat losses). The heat transfer inside the hot liquid zone is described by the usual thermal energy balance, which for a 1-dim. reservoir (Fig. 1) of uniform properties along the vertical ($z-$) direction reads [7]:

$$C\frac{\partial T'}{\partial t'} + U\frac{\partial T'}{\partial x'} = k\frac{\partial^2 T'}{\partial x'^2} + \frac{2k_1}{h}\left(\frac{\partial T_1'}{\partial z'}\right)_{z'=0} \quad (1)$$

and similarly in radial geometries. The second term on the LHS of (1) expresses heat transfer by convection while the last term on the RHS represents the lateral heat losses which couple heat transfer in the hot liquid zone and the surroundings.

In the above, C is the volumetric heat capacity term of the hot liquid zone and U the volumetric velocity of the flowing water–oil mixture.

$$C = \phi_w\rho_w c_{pw} + \phi_o\rho_o c_{po} + \phi_r\rho_r c_{pr},$$

$$U = u_w\rho_w c_{pw} + u_o\rho_o c_{po},$$

$$\phi_w + \phi_o + \phi_r = 1,$$

while subscript 1 refers to quantities of the surrounding formations and superscript $'$ denotes dimensional variables.

1.1. *Heat transfer in the surroundings*

Heat transfer in the under- and over-lying formations proceeds by pure heat conduction coupled to the heat transfer in the reservoir via appropriate conditions [7]. In most practical cases at the prevailing injection rates convection dominates over conduction along the reservoir, and one can reasonably assume that heat flows in the surroundings mainly by 1-dim. conduction along the vertical coordinate z. Clearly, the effectiveness of this approximation depends upon the magnitude of the reservoir Peclet number. For typical conditions of practical interest, the Peclet number is sufficiently large, $O(10^2)$, so that use of the above simplification is justified when calculating the amount of heat transferred from the reservoir to the surroundings (see below and compare also with [3, 7, 8]). The approximation has been tested in both analytical [9] and numerical [1, 6] studies and found very satisfactory in thermal recovery modelling. With this assumption, we can evaluate the lateral heat losses, $-2k_1/h\,(\partial T'_1/\partial z')_{z'=0}$, by considering the heat flux at the origin of a 1-dim., semi-infinite heat conducting medium with surface temperature $T'(x,t)$ and initial temperature T_i. For a continuous and smooth Y' one obtains [7]:

$$-k_1\left(\frac{\partial T'_1}{\partial z'}\right)_{z'=0} = \frac{k_1}{\sqrt{\pi\alpha_1}}\int_0^{t}\frac{\partial T'}{\partial \tau}\frac{d\tau}{\sqrt{(t'-\tau)}} \qquad (2)$$

which gives the local instantaneous heat losses in terms of the temperature history at any point of the reservoir boundaries. The above result is valid for any thermal recovery process and may considerably facilitate heat transfer calculations as compared, for instance, with [10, 11]. Substituting expression (2) into the RHS of (1) we obtain

$$C\frac{\partial T'}{\partial t'} + U\frac{\partial T'}{\partial x'} = k\frac{\partial^2 T'}{\partial x'^2} - \frac{2k_1}{h}\cdot\frac{1}{\sqrt{\pi\alpha_1}}\int_0^{t'}\frac{\partial T'}{\partial \tau}\cdot\frac{d\tau}{\sqrt{(t'-\tau)}} \qquad (3)$$

and similarly for radial geometries [7].

The B.C.'s are:

$$t' = 0, \qquad T' = T_i;$$
$$x' \to \infty, \qquad T' \to T_i;$$
$$x' = \int_0^{t'} v'(\tau)\,d\tau, \qquad T' = T_s.$$

1.2. *The assumption of constant convection*

The second term in the LHS of (3) represents convective heat transfer and is generally a function of both independent and dependent variables, to be determined by a simultaneous solution of the momentum and thermal energy equations. To simplify the analysis further, we introduce the customary approximation that the velocity U in linear geometries (or the velocity $U/2\pi r$ in radial geometries) and the volumetric storage C defined earlier are constant with respect to x' (or r), t', and T'. This approximation is a crucial one, since it enables the consideration of the heat transfer equation independently of the momentum equations. The approximation can be shown [7] to be justified under conditions of: (i) small differences between the volumetric heat capacities of oil and water, (ii) incompressible fluids, and (iii) constant (or slowly varying) volumetric velocities through the front. The first two conditions are actually satisfied in situations of practical interest. The third condition is satisfied at large times (constant) or at small and intermediate times (slowly varying) in processes involving constant injection rates (see [4]). Introducing the dimensionless variables:

$$T = \frac{T' - T_i}{T_s - T_i}, \quad t = \frac{t'}{\left[\left(\frac{C}{\rho_1 c_{p1}}\right)^2 \frac{h^2}{4\alpha_1}\right]},$$

$$x = \frac{x'}{\left(\frac{|U|\,h^2 C}{4k_1\rho_1 c_{p1}}\right)}, \quad v = \frac{Cv'}{|U|}, \quad Pe = \frac{|U|h}{2k_1}\left(\frac{\alpha_1}{\alpha}\right)^{1/2}. \qquad (4)$$

Equation (3) becomes

$$\frac{\partial T}{\partial t} + \frac{U}{|U|}\cdot\frac{\partial T}{\partial x} = \frac{1}{Pe^2}\cdot\frac{\partial^2 T}{\partial x^2} - \frac{1}{\sqrt{\pi}}\int_0^t \frac{\partial T}{\partial \tau}\cdot\frac{d\tau}{\sqrt{(t-\tau)}}, \qquad (5)$$

$$0 < t, \quad \int_0^t v(\tau)\,d\tau \le x$$

with

$$t = 0, \qquad T = 0;$$
$$x \to \infty, \qquad T \to 0;$$
$$x = \int_0^t v(\tau)\,d\tau, \qquad T = 1.$$

A similar equation is obtained for radial geometries [7]. Note that the dimensionless quantity Pe^2 expresses the product of the two ratios:

$$\frac{(\text{convection } x\text{-direction})}{(\text{conduction } x\text{-direction})}, \frac{(\text{convection } x\text{-direction})}{(\text{conduction } z\text{-direction})}.$$

Frequently, $\alpha_1 = \alpha$, thus Pe is the usual Peclet number.

We have now formulated the process of heat transfer in the hot liquid zone in terms of a single linear integro-differential equation (5) which can be handled more easily than a system of two PDE's, particularly in a region with moving boundaries. This equation is generally not amenable to analytical treatment. Particular values of Pe and a certain class of functions $v(t)$, however, permit analytical or asymptotic solutions for the case of linear geometry. In radial geometry analytical solutions are possible only in the limiting situation $Pe \to \infty$. In the following, we will consider those cases that admit analytical solutions. It should be pointed out that although the front velocity in a thermal process is not known *a priori* (and it is actually implicitly determined), most of the assumed profiles in this study have direct practical significance in thermal recovery applications [4].

2. SOLUTION OF THE MOVING BOUNDARY PROBLEM (5)

2.1. Pe ≫ 1 (*negligible horizontal conduction*)

When *Pe* is large, equation (5) takes the simpler form valid for both geometries

$$\frac{\partial T}{\partial t}+\frac{U}{|U|}\cdot\frac{\partial T}{\partial x}=-\frac{1}{\sqrt{\pi}}\int_0^t\frac{\partial T}{\partial \tau}\cdot\frac{d\tau}{\sqrt{(t-\tau)}} \quad (6)$$

in the domain $0 < t$,

$$\int_0^t v(\tau)\,d\tau \leq x$$

with B.C.:

$$t=0, \qquad T=0;$$

$$x\to\infty, \qquad T\to 0;$$

$$x=\int_0^t v(\tau)\,d\tau, \qquad T=1.$$

In the case of radial geometry x denotes πr^2.

In physical terms, equation (6) corresponds to convection-dominated reservoirs (high injection rates) frequently encountered in thermal recovery. For example, a typical linear steam drive involves *Pe* as high as 25, whereas an even higher value obtains for a typical radial steam drive, $Pe = 10\,320$, [7]. This attaches a particular significance to the solutions of (6) in thermal recovery processes that operate under normal injection rates.

The main difficulty associated with the solution of (6) arises as expected from the existence of the moving boundary. To proceed we observe that (6) is composed of a hyperbolic part (LHS) and a sink term of the convolution integral type (RHS). Since the initial condition is $T = 0$, we expect a non-trivial solution to exist if and only if $U > 0$. The non-trivial part of this solution lies in the domain $0 < t$, $x < t$ (Fig. 2a) outside of which $T = 0$. Since the variable x in the region of interest also satisfies

$$\int_0^t v(\tau)\,d\tau \leq x$$

a non-trivial solution exists only if

$$\int_0^t v(\tau)\,d\tau < t. \quad (7)$$

This constraint states that heat transfer can only occur if the convective heat wave travels faster than the moving boundary. Introducing the new variables

$$\theta = t - x, \quad \chi = x, \quad \Theta(\theta,\chi) = T(t,x) \quad (8)$$

the region of integration becomes (Fig. 2b) $\theta > 0$, $\chi > \Gamma(\theta)$ where $\chi = \Gamma(\theta)$ is the image of the curve

$$x = C(t) = \int_0^t v(\tau)\,d\tau,$$

under the above transformation. Thus, $\Gamma(\theta)$ is implicitly defined by

$$\chi = \int_0^{\theta+\chi} v(\tau)\,d\tau \quad (9)$$

and the corresponding boundary velocity by

$$\omega(\theta)=\frac{d\chi}{d\theta}=\frac{v(t)}{1-v(t)}. \quad (10)$$

In the new coordinate system the integro-differential equation takes the form:

$$\frac{\partial\Theta}{\partial\chi}=-\frac{1}{\sqrt{\pi}}\int_0^\theta\frac{\partial\Theta}{\partial\tau}\cdot\frac{d\tau}{\sqrt{(\theta-\tau)}},$$

$$0<\theta, \quad \Gamma(\theta)\leq\chi \quad (11)$$

with B.C.: $\theta = 0$, $\Theta = 0$;

$$\chi=\int_0^\theta \omega(\tau)\,d\tau, \quad \Theta = 1.$$

We now claim that the moving boundary problem (11) is equivalent to the pure heat conduction problem

$$\frac{\partial\Theta}{\partial\theta}=\frac{\partial^2\Theta}{\partial\chi^2} \quad (12)$$

with I.C. $\theta = 0$, $\Theta = 0$
and B.C. $\chi\to\infty$, $\Theta\to 0$;

$$\chi=\int_0^\theta \omega(\tau)\,d\tau, \quad \Theta = 1.$$

Indeed, by taking the Laplace Transform of (11), (12), we see that, within a multiplicative factor, $A(s)$, to be determined from the moving boundary condition, both equations give rise to the same transformed expression:

$$A(s)\exp(-\chi\sqrt{s}).$$

The uniqueness of the inverse Laplace Transform guarantees that (12) with its boundary conditions have the same solution (see also [12]). The moving boundary problem (12) can be solved by a variety of numerical and in some cases analytical techniques. Analytical solution can be pursued by employing the moving coordinates

$$\theta, \ \xi = \chi - \int_0^\theta \omega(\tau)\,d\tau,$$

thus immobilizing the moving origin (Fig. 2c)

$$\frac{\partial\Theta}{\partial\theta}-\omega(\theta)\frac{\partial\Theta}{\partial\xi}=\frac{\partial^2\Theta}{\partial\xi^2} \quad (13)$$

with B.C.:

$$\theta = 0, \quad \Theta = 0;$$

$$\xi\to\infty, \quad \Theta\to 0;$$

$$\xi = 0, \quad \Theta = 1.$$

Equation (13) admits analytical solutions for certain classes of functions $\omega(\theta)$. As previously indicated we will consider cases that admit closed form solutions while at the same time have practical application to thermal recovery. It is understood that numerical techniques can be used to solve equation (16) for arbitrary front velocity, $\omega(\theta)$.

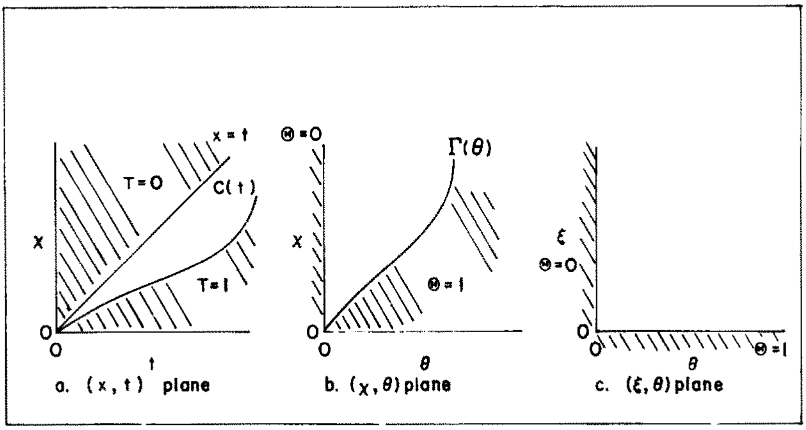

FIG. 2. Regions of integration for equation (6).

2.1.1. *Fixed boundary.* This is typical of hot water injection (hot-waterflood). Here $v(t) \equiv 0$ and, by (10), $\omega(\theta) \equiv 0$. Also $\xi = \chi$ and (13) reads:

$$\frac{\partial \Theta}{\partial \theta} = \frac{\partial^2 \Theta}{\partial \xi^2}. \tag{14}$$

The solution of this problem is

$$\Theta(\theta,\xi) = \operatorname{erfc}\left\{\frac{\xi}{2\sqrt{\theta}}\right\} \cdot H(\theta)$$

or in the original variables

$$T(t,x) = \operatorname{erfc}\left\{\frac{x}{2\sqrt{(t-x)}}\right\} \cdot H(t-x), \tag{15}$$

where $H(t)$ is the Heaviside step function. Figure 3 shows temperature profiles for various times.

A solution identical to (15) has been obtained by Lauwerier [8] by a more complicated approach. The present method is similar and can be extended to cases with different boundary conditions. For example, when the boundary temperature is varying, $g(t)$, one can easily derive by superposition:

$$T(t,x) = \frac{x}{2\sqrt{\pi}} \int_0^t \exp\left\{-\frac{x^2}{4\tau}\right\} \times \frac{g(t-\tau)}{\tau^{3/2}} \mathrm{d}\tau \cdot H(t-x). \tag{16}$$

2.1.2. *Constant front velocity.* A second interesting case concerns condensation fronts advancing with constant velocity. Such situations are encountered in thermal processes [7] at early and intermediate times (e.g. steam injection) or at large times (e.g. combustion) where the front velocity may be assumed to be constant (or "slowly varying"), $v(t) = c$. In the absence of horizontal conduction, constraint (7) dictates $c < 1$ and in dimensional variables

$$Cv < U. \tag{17}$$

From (10), $\omega(\theta) = c/(1-c) > 0$. The curve $C: x = ct$ maps onto Γ: $\chi = \theta\, c/(1-c)$, hence $\xi = \chi - \theta\, c/(1-c)$ and the heat transfer equation reads:

$$\frac{\partial \Theta}{\partial \theta} - \frac{c}{(1-c)} \frac{\partial \Theta}{\partial \xi} = \frac{\partial^2 \Theta}{\partial \xi^2}, \quad 0 < \theta, \quad 0 < \xi. \tag{18}$$

The solution of (18) in terms of the original variables is [7]:

$$T(t,x) = \frac{1}{2}\left\{\operatorname{erfc}\left(\frac{x}{2\sqrt{(t-x)}}\right) + \exp\left\{-\frac{c}{(1-c)^2}(x-ct)\right\} \times \operatorname{erfc}\left[\frac{x(1+c)-2ct}{2(1-c)\sqrt{(t-x)}}\right]\right\} H(t-x),$$
$$0 < t, \quad ct < x. \tag{19}$$

Equation (19) provides a closed form expression for the temperature distribution in a convection-dominated hot liquid zone bounded by a front advancing with constant velocity. Figure 4 shows various profiles of T vs x for various t and c. As $t \to \infty$, $T(t,x)$ approaches

$$T(t,x) = \exp\left\{-\frac{c}{(1-c)^2}(x-ct)\right\} H(t-x) \tag{20}$$

which has the form of a wave travelling with a velocity equal to the velocity of the moving front. The

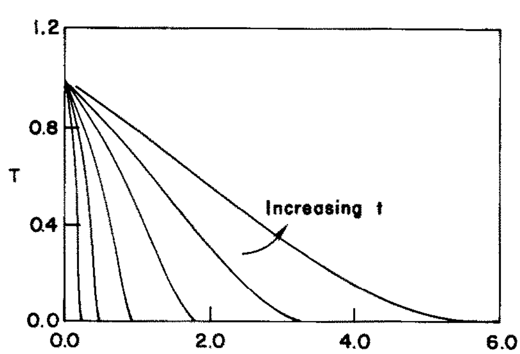

FIG. 3. Temperature profiles of equation (15) for $t = 0.25, 0.5, 1.0, 2.0, 4.0, 8.0$.

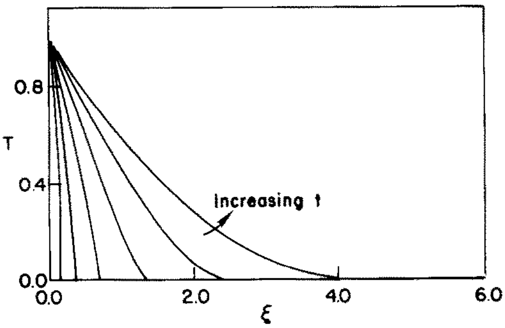

FIG. 4(a). Temperature profiles of equation (19) for $c = 0.2$ and $t = 0.25, 0.5, 1.0, 2.0, 4.0, 8.0$.

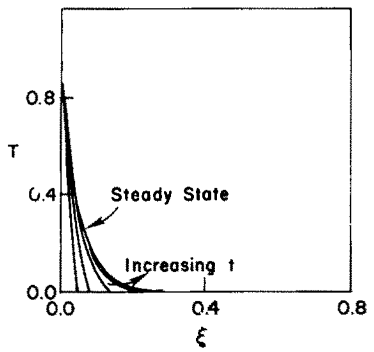

FIG. 4(b). Temperature profiles of equation (19) for $c = 0.8$ and $t = 0.25, 0.5, 1.0, 2.0, 4.0, 8.0$.

characteristic time for approaching this asymptotic value is in dimensional variables equal to

$$\left[\frac{(1-c)}{c^2}\left(\frac{C}{\rho_1 c_{p1}}\right)^2 \frac{h^2}{\alpha_1}\right];$$

thus, the approach to the quasi-steady state is controlled solely by the front velocity and not by the net convective heat flux (as for example in the case of zero lateral heat losses [7]).

Similar results are expected when the time-scale of change in the velocity profile of the advancing front is relatively large compared to the above characteristic time (slowly varying front velocities). In such cases one can use the quasi-steady state solution (20) with c a function of time to describe the temperature distribution in the hot liquid zone. This quasi-steady state approximation can be applied to a variety of thermal recovery calculations [4, 7] and to other physical problems [13].

The applicability of the above results is restricted by constraint (7) requiring a positive net convective heat flux through the front. This constraint may be violated at early and intermediate times of a thermal process (e.g. steam injection at constant injection rates). However, the difficulty can be circumvented by including horizontal conduction in the description of heat transfer, as shown below.

For future reference we consider the behavior of (19) for small values of c. The dimensionless conductive heat flux in this case approaches the front velocity c:

$$-\left.\frac{\partial T}{\partial x}\right|_{x=ct} \sim c + \frac{1}{\sqrt{\pi t}}\exp\left\{-\frac{c^2 t}{4}\right\} - \frac{c}{2}\operatorname{erfc}\left\{\frac{c\sqrt{t}}{2}\right\} \quad (21)$$

[cf. equation (44)].

2.1.3. *Front velocity of the form* $v(t) = a/\sqrt{(t + a^2)}$. The class of fronts characterized by the velocity expression $v = a/\sqrt{(t + a^2)}$, $a > 0$, is a third case of considerable interest, for it can fairly accurately describe the front velocity at large times in a number of thermal recovery processes. For example, it was shown [4] that in steam injection the steam front velocity approaches relatively rapidly the asymptotic form $v \sim a/\sqrt{t}$, where a is fixed by the ratio of latent to the total heat injected. As in the two previous cases, this velocity profile leads to an analytical solution.

In the present case, the curve C described by: $x = -2a^2 + 2a\sqrt{(t + a^2)}$, maps onto $\Gamma: \chi = 2a\sqrt{\theta}$. From (10) we obtain $\omega(\theta) = a/\sqrt{\theta}$ and $\xi = \chi - 2a\sqrt{\theta}$. Note that the constraint (7) is satisfied for all $a > 0$. The subsidiary equation (13)

$$\frac{\partial \Theta}{\partial \theta} - \frac{a}{\sqrt{\theta}} \cdot \frac{\partial \Theta}{\partial \xi} = \frac{\partial^2 \Theta}{\partial \xi^2} \quad (22)$$

admits a similarity solution in terms of $\eta = \xi/2\sqrt{\theta} + a$. In the original variables, one obtains

$$T(t,x) = \operatorname{erfc}\left\{\frac{x}{2\sqrt{(t-x)}}\right\}\Big/\operatorname{erfc} a \cdot H(t-x) \quad (23)$$

$$\text{on } -2a + 2a\sqrt{(t + a^2)} < x$$

Expression (23) is identical within a multiplicative constant with Lauwerier's solution, (15), obtained for fixed fronts. In Fig. 5 we show typical T profiles for various times and values of a. Variations in a have a significant effect. As a increases, the temperature profiles become steeper, due to the increased front velocity. The conductive heat flux at the origin is

$$-\left.\frac{\partial T}{\partial x}\right|_{x\in C} = \frac{e^{-a^2}}{\sqrt{\pi}\operatorname{erfc} a} \times \left\{\frac{t - a\sqrt{(t + a^2)} + a^2}{(t - 2a\sqrt{(t + a^2)} + 2a^2)^{3/2}}\right\} \quad (24)$$

and has the asymptotic behavior

$$\left.\frac{\partial T}{\partial x}\right|_{x\in C} = \frac{1}{(\pi t)^{1/2}} \frac{e^{-a^2}}{\operatorname{erfc} a}, \quad t \to \infty.$$

This asymptotic expression also applies to the problem including horizontal conduction for the case $Pe = 1$ (see section 2.2.3).

Equations (23) and (24) and particularly their asymptotic limits were used in determining the behavior of the steam front velocity at large times in 1-dim. steam injection [4].

2.2. *Finite* Pe (*includes horizontal conduction*)

As previously noted, constraint (7) restricts some-

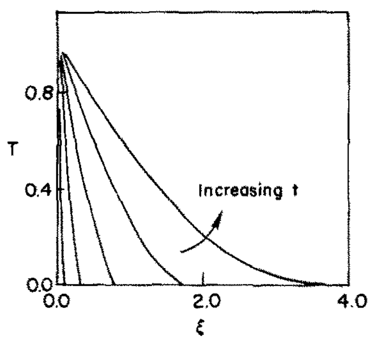

FIG. 5(a). Temperature profiles of equation (23) for $a = 0.5$ and $t = 0.25, 0.5, 1.0, 2.0, 4.0, 8.0$.

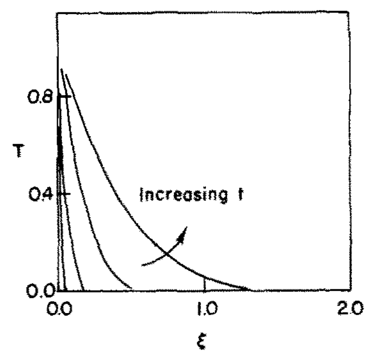

FIG. 5(b). Temperature profiles of equation (23) for $a = 1.5$ and $t = 0.25, 0.5, 1.0, 2.0, 4.0, 8.0$.

what the applicability of the above results. To include processes whose parameters do not satisfy (7) we proceed with the study of eqn. (5) for finite Pe. Such an analysis will provide an estimate of the contribution of horizontal conduction in the reservoir at low injection rates. As before, we regard the heat transfer in the surroundings to be dominated by vertical conduction alone. The addition of horizontal conduction does not permit a uniform representation of linear and radial geometries (in contrast to the previous case); therefore, we will confine our investigation to linear reservoirs, always keeping in mind that the asymptotic results as $Pe \to \infty$ carry over a radial geometries. The cases to be examined include fixed fronts with arbitrary Pe, steady state profiles for fronts of constant velocity with arbitrary Pe, and a typical case of low injection rates ($Pe = 1$).

2.2.1. *Fixed boundary, arbitrary* Pe. Equation (5) with a fixed boundary is a more realistic representation of hot water injection than the problem discussed in 2.1.1. The solution to the present problem can be obtained by a direct application of the Laplace transformation with the result:

$$T(t,x) = \frac{xPe}{2\sqrt{\pi}} \int_0^t \exp\left[-\frac{Pe^2\tau}{4}\left(1 - \frac{xU}{\tau|U|}\right)^2\right] \times \operatorname{erfc}\left\{\frac{\tau}{2\sqrt{(t-\tau)}}\right\} \frac{d\tau}{\tau^{3/2}}. \quad (25)$$

This is similar to the expression obtained by Avdonin [14] by different means. Equation (25) is valid for arbitrary U, in contrast with the solution developed in 2.1.1 where the condition $U > 0$ was necessary for the existence of a non-trivial solution. The latter restriction, of course, is of minor practical significance.

The integral in (25) simplifies considerably when $Pe = 1$ to yield:

$$T(t,x) = \operatorname{erfc}\left(\frac{x}{2\sqrt{t}}\right), \qquad U < 0 \quad (26)$$

$$T(t,x) = e^{-x} \operatorname{erfc}\left(\frac{x}{2\sqrt{t}}\right) \qquad U < 0 \quad (27)$$

(see also 2.2.3.1). In Fig. 6(a) we plot profiles of T for different times and $Pe = 1$ in the case $U > 0$. As illustrated by Fig. 6(b) the profiles obtained from (26) are close to Lauwerier's profiles (15) except for large x, when the heat wave penetrates farther into the unheated zone. As Pe increases, however, the two profiles approach each other and as $Pe \to \infty$ they eventually coincide. The convergence is faster for higher values of Pe. It follows that the Lauwerier solution is adequate when $Pe \gg 1$. On the other hand, for low injection rates, Pe is of the order of unity and solutions given by equations (25) or (26) are more accurate.

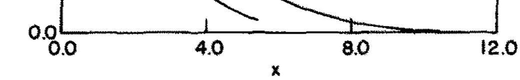

FIG. 6(a). Temperature profiles of equation (26) for $U > 0$, $Pe = 1$ and $t = 0.25, 0.5, 1.0, 2.0, 4.0, 8.0$.

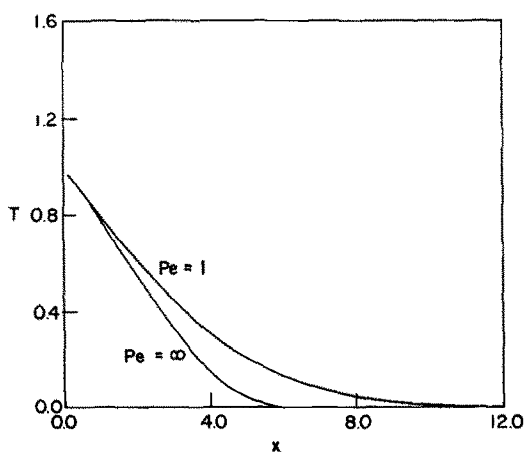

FIG. 6(b). Temperature profiles of equation (26) for $U > 0$, $Pe = 1$, (upper curve) and equation (15) (lower curve), at $t = 8.0$.

2.2.2. *Constant front velocity, asymptotic solutions for arbitrary* Pe. The inclusion of horizontal conduction in the case of a constant velocity boundary allows for the derivation of explicit but rather cumbersome expressions. To simplify the discussion we elect to study the simpler problem of asymptotic solutions for large times. Introducing the moving coordinates t, $\xi = x - ct$, we rewrite (5) and the B.C. as:

$$\frac{\partial T}{\partial t} + \left(\frac{U}{|U|} - c\right)\frac{\partial T}{\partial \xi} = \frac{1}{Pe^2}\cdot\frac{\partial^2 T}{\partial \xi^2} - \frac{1}{\sqrt{\pi}}\int_0^t \frac{\partial T}{\partial \tau}[\tau, \xi + (t-\tau)c]\frac{d\tau}{\sqrt{(t-\tau)}} + \frac{c}{\sqrt{\pi}}\int_0^t \frac{\partial T}{\partial \xi}[\tau, \xi + (t-\tau)c]\frac{d\tau}{\sqrt{(t-\tau)}}, \quad (28)$$

$$t = 0, \quad T = 0;$$
$$\xi \to \infty, \quad T \to 0;$$
$$\xi = 0, \quad T = 1;$$

where c is the velocity of the boundary. As $t \to \infty$, equation (28) becomes

$$\left(\frac{U}{|U|} - c\right)\frac{\partial T}{\partial \xi} = \frac{1}{Pe^2}\cdot\frac{\partial^2 T}{\partial \xi^2} + \frac{c}{\sqrt{\pi}} \lim_{t\to\infty}\int_0^t \frac{\partial T}{\partial \xi}[\xi + c(t-\tau)]\cdot\frac{dt}{\sqrt{(t-\tau)}}, \quad (29)$$

or after rearrangement

$$\left(\frac{U}{|U|} - c\right)\frac{\partial T}{\partial \xi} = \frac{1}{Pe^2}\cdot\frac{\partial^2 T}{\partial \xi^2} + \frac{\sqrt{c}}{\sqrt{\pi}}\int_0^\infty \frac{\partial T}{\partial \xi}(\xi + \sigma)\frac{d\sigma}{\sqrt{\sigma}}, \quad (30)$$

with B.C.:

$$\xi \to \infty, \quad T \to 0;$$
$$\xi = 0, \quad T = 1.$$

The solution of this integro-differential equation in one independent variable provides the asymptotic solutions. In constrast to (17), c is not bound by any constraint.

Introducing the notation:

$$x = \xi, \quad \phi(x) = \frac{\partial T}{\partial \xi},$$

$$\lambda = \frac{\sqrt{c}}{\left(\frac{U}{|U|} - c\right)}, \quad \mu = \frac{1}{Pe^2\left(\frac{U}{|U|} - c\right)},$$

where $c \neq U/|U|$ and specifying without loss in generality $c > 0$, we obtain

$$\phi(x) = \mu\phi'(x) + \frac{\lambda}{\sqrt{\pi}}\int_0^\infty \phi(\sigma + x)\frac{d\sigma}{\sqrt{\sigma}}. \quad (31)$$

In Appendix A we show that equation (31) can be reduced to the ODE

$$\phi'''(x) - \frac{2}{\mu}\phi''(x) + \frac{1}{\mu^2}\phi'(x) + \frac{\lambda^2}{\mu^2}\phi(x) = 0 \quad (32)$$

which admits the general solution

$$\phi(x) = A_1 \exp(z_1 x) + A_2 \exp(z_2 x) + A_3 \exp(z_3 x) \quad (33)$$

where z_1, z_2, z_3 are the roots of:

$$z\left(z - \frac{1}{\mu}\right)^2 + \frac{\lambda^2}{\mu^2} = 0. \quad (34)$$

Returning to the original notation and integrating, we get a physically acceptable solution,

$$T(\xi) = \exp(z_1 \xi) \quad (35)$$

where z_1 is the real negative root of (34). Values of z_1 for various values of Pe, c are shown in Tables 1 and 2. As expected, the larger Pe and/or c is, the steeper the temperature gradient at the moving origin. The results obtained should agree with the solutions discussed in 2.1.2. Indeed, when $Pe \to \infty$, then $\mu \to 0$ and (34) admits the unique solution $z = -\lambda^2$, hence

$$T(\xi; Pe \to \infty) = \exp\left[-\frac{c}{(1-c)^2}\xi\right] \quad (36)$$

which is identical to expression (20). In Fig. 7 we show asymptotic profiles for various Pe. As Pe increases, at constant $c < 1$, the temperature profiles approach the solution (20) which was obtained by neglecting horizontal conduction. The agreement is better for large Pe and small c (Table 2). On the other hand, for small values of Pe (Table 2), the difference between the gradients given by equations (20) and (35) is significant and should be properly taken into account when designing a thermal recovery process at low injection rates.

The profiles (35) can be used to describe the temperature distribution in thermal processes in which the front has a constant (or "slowly" varying) velocity and the injection rates are low [$Pe = O(1)$], or the injection parameters do not satisfy constraint (7). In this sense, they complement the quasi-steady results obtained in 2.1.2 for the case of negligible horizontal conduction (high injection rates). An application of this quasi-steady state approximation in estimating the velocity of the steam front at early and intermediate times of a steam drive was presented in [4]. The applicability of the approximation clearly requires that the characteristic time of approach to the asymptotic solution is small enough compared to the time-scale of change of the front velocity. To obtain an estimate of the latter characteristic time one could rely on the analysis presented in 2.1.2 ($Pe \to \infty$) and on the following discussion of the special case $Pe = 1$. The desired estimate for other values of Pe is expected to lie between the results corresponding to the two extreme cases $Pe = \infty$, $Pe = 1$. One can then develop a criterion that defines the region of validity of this quasi-steady state approximation [4].

Table 1. Values of z_l for Various Values of Pe, c

	$c = 0.1$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.00
$Pe = 10$	−0.12311	−0.31009	−0.60185	−1.0724	−1.8591	−3.2130	−5.5414	−9.3114	−14.7236	−25.5443
$Pe = 31.7$	−0.12342	−0.31225	−0.61117	−1.1070	−1.9842	−3.6819	−7.4074	−16.9905	−43.627	−100.0000
$Pe \to \infty$	−0.12345	−0.31250	−0.61224	−1.1111	−2.0000	−3.7500	−7.7777	−20.0000	−90.000	$-\infty$

Table 2. Values of z_l for Various Values of Pe, c

c	Pe	z_l
100	10	−9910
10	3.17	−993
	1	−100
	0.317	−10.21
1	10	−910
	3.17	−93
	1	−10
	0.317	−1.11
0.1	10	−4.64
	3.17	−4.64
	1	−1
	0.317	−0.21
	10	−0.1231
	3.17	−0.1231
	1	−0.1
	0.317	−0.050
0.1	$Pe \to \infty$	−0.1234

We complete the study of heat transfer in the hot liquid zone by considering a special case ($Pe = 1$) which leads to exact solutions and may be considered as representative of thermal recovery processes at low injection rates.

2.2.3. Pe = 1, *arbitrary velocity*. When $Pe = 1$ the integro-differential equation (5) admits a simple solution. To show this we take the Laplace transformation of (5) with respect to time. The resulting expression is generally very complicated but for $Pe = 1$ it assumes the simpler form

$$L_t\{T\} = A(s)\exp\left\{\left[\frac{1}{2}\left(\frac{U}{|U|} - 1\right) - \sqrt{s}\right]x\right\} \tag{37}$$

where $A(s)$ is an unknown function to be determined from the boundary conditions. When $U > 0$ (37) becomes

$$L_t\{T\} = A(s)\exp\left[-x\sqrt{s}\right] \tag{38}$$

which implies that $T(t,x)$ satisfies the pure heat conduction equation

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}. \tag{39}$$

In other words, in the special case $Pe = 1$, the lateral heat losses are exactly balanced by the convective heat flux and, as a result, heat transfer is governed by pure heat conduction along the horizontal coordinate x. On the other hand, when $U < 0$ (e.g. reverse combustion), (37) implies

$$\frac{\partial T}{\partial t} - 2\frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial x^2} + T. \tag{40}$$

Before exploring various special cases of boundary motion, we restate the conditions under which the interesting case $Pe = 1$ arises. By definition, $Pe = 1$ implies $|U| = 2k_1/h\sqrt{(\alpha/\alpha_1)}$, which for the usual case $\alpha = \alpha_1$ shows, in a qualitative sense, why convection counterbalances heat losses to the surrounding formations.

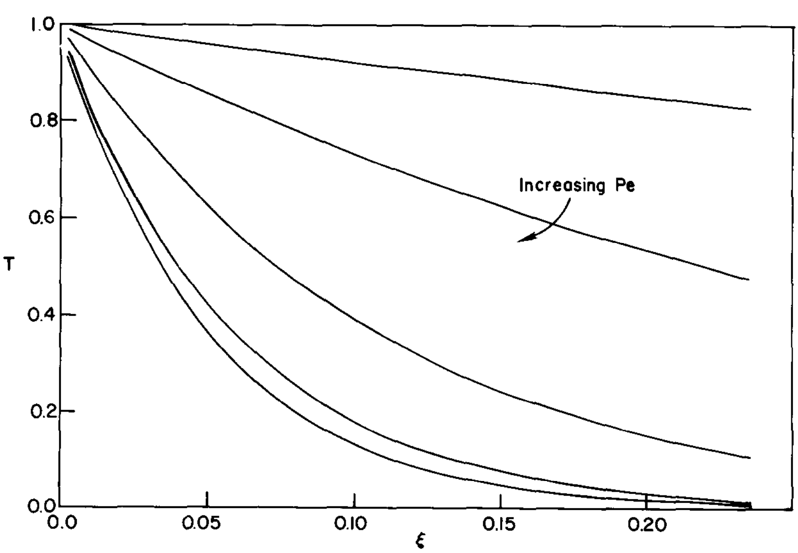

FIG. 7. Temperature profiles of equation (36) for Pe = 1, 3.17, 10, 31.7 (upper four curves), and of equation (20) for $c = 0.1$ (lower curve).

Fixed boundary. This problem has been discussed in 2.2.1 for the case of arbitrary Pe. The results of that section take a particularly simple form when $Pe = 1$. In light of the above analysis one can easily deduce that expressions (26), (27) are the solutions of equations (39) and (40).

Constant front velocity. When $v(t) = c$ we can use the moving coordinates $t, \xi = x - ct$ to obtain the familiar equation

$$\frac{\partial T}{\partial t} - c\frac{\partial T}{\partial \xi} = \frac{\partial^2 T}{\partial \xi^2}, \quad U > 0 \tag{41}$$

which by direct application of the Laplace transformation yields:

$$T = \frac{1}{2}\left\{\text{erfc}\left(\frac{x}{2\sqrt{t}}\right) + \exp\left[-c(x - ct)\right] \times \text{erfc}\left(\frac{x - 2ct}{2\sqrt{t}}\right)\right\}, \; ct \le x. \tag{42}$$

In the limit of large times this reduces to the steady state solution:

$$T = \exp\left[-c(x - ct)\right] \tag{43}$$

in accordance with (35). The conductive heat flux through the origin

$$-\left.\frac{\partial T}{\partial x}\right|_{x=ct} = c + \frac{1}{\sqrt{\pi t}}\exp\left[-\frac{c^2 t}{4}\right] - \frac{c}{2}\text{erfc}\left(\frac{c\sqrt{t}}{2}\right) \tag{44}$$

tends asymptotically to $-c$. One can accordingly estimate the characteristic time of convergence to be of the order of

$$\left[\frac{1}{c^2}\left(\frac{C}{\rho_1 c_{p1}}\right)^2 \frac{h^2}{\alpha_1}\right],$$

in dimensional variables. This time is inversely proportional to the square of the boundary velocity c, just as in the case $Pe \to \infty$ (see 2.1.2). The characteristic times for $Pe = 1$ and $Pe = \infty$ are close to each other when c is small but differ considerably when c approaches 1. No comparison is possible when $c > 1$ since the analysis in 2.1.2 applies only when $c < 1$.

A comparison of the heat flux given by (44) with that given by (21) which corresponds to $Pe \to \infty$ (no horizontal conduction) is of some interest. When the front velocity is small, the two expressions are identical and horizontal conduction has almost no effect on the conductive heat flux through the steam front despite the low injection rates.

Front velocity $v(t) = a/\sqrt{t}$. In the content of thermal recovery applications it would be useful to derive the temperature distribution for cases where the characteristic times for changes in the front velocity and the heat transfer are comparable. Analytical solutions in this case are possible when the front velocity has the form

$$v(t) = a/\sqrt{t}$$

where a is an arbitrary constant. The practical importance of these fronts in 1-dim. steam injection has been emphasized in 2.1.3. and is further discussed in [4].

In moving coordinates t, $\xi = x - 2a\sqrt{t}$ the heat equation (39) becomes

$$\frac{\partial T}{\partial t} - \frac{a}{\sqrt{t}}\cdot\frac{\partial T}{\partial \xi} = \frac{\partial^2 T}{\partial \xi^2}$$

the solution of which is obtained as before

$$T(t,x) = \text{erfc}\left\{\frac{x}{2t^{1/2}}\right\}\Big/\text{erfc}\, a \tag{45}$$

with a dimensionless conductive flux at the origin

$$-\left.\frac{\partial T}{\partial x}\right|_{x=2a\sqrt{t}} = \frac{1}{\sqrt{\pi t}}\cdot\frac{\exp(-a^2)}{\text{erfc}\, a} \tag{46}$$

A remarkable similarity between expressions (46) and (24), ($Pe \to \infty$), exists in the limit of large times. As t increases, the front velocity in 2.1.3 approaches $a/\sqrt{t}$ while the dimensionless conductive heat flux in (24) approaches that of (46). The implication of this result is quite significant: it shows that in cases where the front velocity takes the asymptotic form $a/\sqrt{t}$ the heat transfer is not appreciably influenced by the inclusion of horizontal conduction. This conclusion is expected to hold for any value of Pe in the range $[1,\infty]$. On the other hand, one should be careful not to discount the effect of horizontal conduction on the rate of convergence to the asymptotic state. It turns out that the larger the value of Pe, the faster the convergence to this state, while for $Pe = 1$ the convergence is very slow and the asymptotic state cannot be attained within practically realistic times.

3. CONCLUSIONS

(1) Heat transfer in the hot liquid zone of a 1-dim. reservoir undergoing a thermal recovery process involving a moving condensation front can be described by a single integro-differential equation with a moving boundary. Closed form expressions for the temperature distribution can be derived for the following cases:

(a) When the dimensionless parameter Pe, that characterizes the importance of horizontal convection relative to horizontal conduction, is large (high injection rates). Analytical solutions have been obtained for a fixed front, a constant velocity front and a front moving according to the form $a/\sqrt{(t + a^2)}$. The resulting expressions are useful in modelling hot waterflood, steam injection and combustion (both forward and reverse) under high (typical) injection rates.

(b) When horizontal conduction cannot be neglected (low injection rates), quasi-steady state temperature profiles have been derived for a front moving at constant velocity. Together with the quasi-steady profiles for $Pe \to \infty$ these expressions are useful in describing the heat transfer in the hot liquid zone at early and intermediate times.

(c) For the special case $Pe = 1$ representative of low injection rates particular solutions have been obtained for fixed boundary, constant front velocity and front velocity equal to $a/\sqrt{t}$. Possible applications include hot waterflood, steam injection or combustion, at low injection rates.

(2) When the front velocity does not assume the above profiles, analytical solutions are generally unattainable. However, the integro-differential equation describing heat transfer can still be reduced to the heat conduction equation with a moving boundary in the special cases $Pe \to \infty$, $Pe = 1$, which can be treated by well established numerical techniques.

(3) Comparison of the results obtained in the various cases above indicates that:

(a) Under normal injection conditions $Pe \gg 1$, the effect of horizontal conduction is insignificant. Thus, it can be neglected in calculations, provided that the operational parameters satisfy the constraint (10).

(b) Even at low injection rates, horizontal conduction makes a negligible contribution under the conditions of small and constant velocity or at large times in the case of fronts asymptotically behaving as $\alpha/\sqrt{t}$.

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APPENDIX A

Introducing the Weyl transform (Erdelyi *et al.* [15]) of order $\frac{1}{2}$ defined by

$$W^{1/2}\{\phi\} \equiv \frac{1}{\sqrt{\pi}} \int_x^\infty \frac{\phi(\sigma)\,d\sigma}{\sqrt{(\sigma - x)}}$$

we write equation (31) as

$$\phi(x) = \mu\phi'(x) + \lambda W^{1/2}\{\phi(x)\}. \tag{A.1}$$

Taking the $W^{1/2}$ transform of (A.1) and rearranging yields

$$W^{1/2}\{\phi(x)\} = \mu W^{1/2}\{\phi'(x)\} + \lambda \int_x^\infty \phi(\xi)\,d\xi, \tag{A.2}$$

$$\phi(x) - \mu\phi'(x) = \lambda\mu W^{1/2}\{\phi'(x)\} + \lambda^2 \int_x^\infty \phi(\xi)\,d\xi. \tag{A.3}$$

Using the identity

$$\frac{d}{dx} W^{1/2}\{\phi(x)\} = W^{1/2}\{\phi'(x)\} \tag{A.4}$$

in (A.1) and combining with (A.3) we obtain

$$\phi(x) - \mu\phi'(x) = \mu[\phi'(x) - \mu\phi''(x)] + \lambda^2 \int_x^\infty \phi(\xi)\,d\xi. \tag{A.5}$$

After differentiation

$$\phi'''(x) - \frac{2}{\mu}\phi''(x) + \frac{1}{\mu^2}\phi'(x) + \frac{\lambda^2}{\mu^2}\phi(x) = 0 \tag{A.6}$$

which is the desired ordinary differential equation.

TRANSFERT THERMIQUE DEVANT DES FRONTS DE CONDENSATION MOBILES DANS LES PROCEDES D'EXTRACTION THERMIQUE DE L'HUILE

Résumé—On étudie le transfert thermique dans la zone liquide qui précède un front mobile de condensation dans un réservoir monodimensionnel d'huile lors d'un procédé d'extraction thermique par courant d'eau chaude, ou par injection de vapeur d'eau ou par combustion in situ. On développe un modèle qui traite du transfert thermique par conduction et convection horizontales dans le réservoir et par conduction verticale (conjugée) autour du réservoir pour tenir compte des pertes de chaleur latérales. Le modèle formulé par une équation intégro-différentielle qui conduit à une représentation intégrale des pertes thermiques latérales pour une région à frontière mobile. En fonction de la grandeur du nombre de Peclet, de la vitesse du front, plusieurs expressions analytiques décrivent la distribution de température dans la zone liquide chaude. La discussion insiste sur le cas $Pe \gg 1$ (grands débits injectés) et $Pe = 1$ (faibles débits injectés). Le cas de *Pe* quelconque est traité par une approximation d'état quasi-stationnaire.

WÄRMEÜBERGANG VOR WANDERNDEN KONDENSATIONSFRONTEN IN THERMISCHEN ÖLGEWINNUNGSPROZESSEN

Zusammenfassung—Dieser Bericht befaßt sich mit der Wärmeübertragung im Flüssigkeitsgebiet vor einer fortschreitenden Kondensationsfront in einem eindimensionalen Ölreservoir bei thermischen Förderprozessen wie Heißwasserflutung, Dampfinjektion oder Verbrennung vor Ort. Ein mathematisches Modell wurde entwickelt, mit dem im Falle der Wärmeübertragung durch horizontale Wärmeleitung und Konvektion im Reservoir sowie durch vertikale (konjugierte) Wärmeleitung in die Umgebung der Formation des Reservoirs die seitlichen Wärmeverluste bestimmt werden können. Das Modell wurde in Form einer Integral-Differentialgleichung dargestellt, welche die integrale Darstellung der seitlichen Wärmeverluste für ein Gebiet mit beweglicher Grenze wiedergibt. Entsprechend der Peclet-Zahl und der Geschwindigkeit der wandernden Front wurden zahlreiche analytische Ausdrücke, die die Temperaturverteilung in der heißen Flüssigkeitszone beschreiben, erhalten. Es werden insbesondere die Fälle $Pe \gg 1$ (hohe Injektionsgeschwindigkeiten) und $Pe = 1$ (niedrige Injektionsgeschwindigkeiten) betrachtet. Der Fall beliebiger *Pe*-Zahlen wird näherungsweise quasistationär behandelt.

ТЕПЛОПЕРЕНОС ПЕРЕД ДВИЖУЩИМСЯ ФРОНТОМ КОНДЕНСАЦИИ В ПРОЦЕССАХ ТЕПЛОВОЙ РЕГЕНЕРАЦИИ МАСЕЛ

Аннотация — Исследуется теплоперенос в жидкости перед фронтом конденсации при его одномерном движении в резервуаре с маслом в процессе тепловой регенерации под действием потока горячей воды, вдува пара или локального горения. Предложена модель, учитывающая перенос тепла теплопроводностью и конвекцией в горизонтальном направлении в резервуаре и теплопроводностью в вертикальном направлении в окружающей резервуар среде, которая позволяет рассчитать боковые потери тепла. Модель основана на интегро-дифференциальном уравнении, в котором используется интегральное выражение для боковых потерь тепла в области с движущейся границей. В зависимости от величины числа Пекле и скорости движения фронта выведены различные аналитические выражения, описывающие распределение температур в нагретой жидкости. Особое внимание уделено случаям $Pe \gg 1$ (большая скорость вдува) и $Pe = 1$ (малая скорость вдува). Случай произвольного значения числа *Pe* анализируется с помощью квазистационарного приближения.